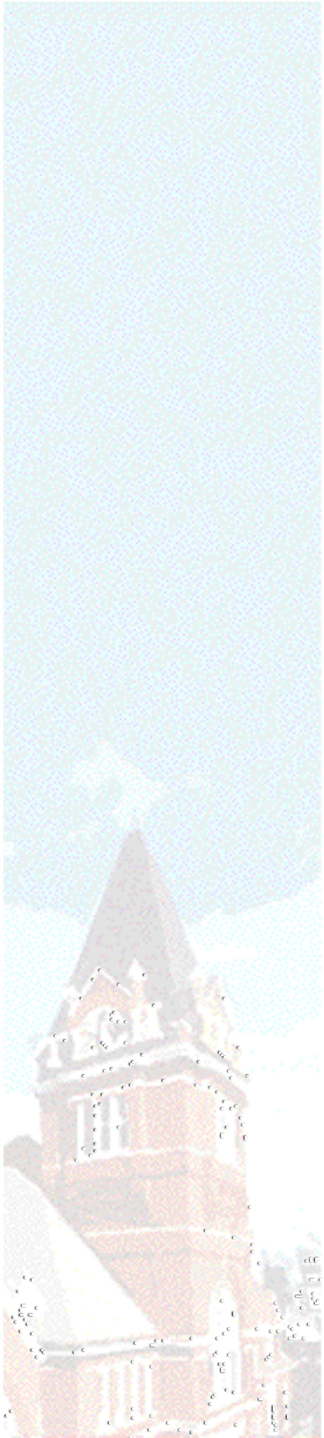

Empirical Receivers for Knowledge-Assisted STAP

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Detectors for STAP

- Typical design approach:
  - Assume a model for data
  - Design an optimal detector
- Statistical modeling of clutter and interference is a hard problem.
- Characteristics can vary greatly from one situation to another.
- Using an explicit model limits flexibility by imposing constraints on expected data structure.
- Performance can degrade significantly when data does not match the model.

Universal Classification

Seminal work by Ziv ('88) and Gutman ('89)

- Theoretical basis: 'method of types' and universal source codes
  - Based on empirical statistics of observed data
  - Independent of source statistics
  - Performance improves with more data
- Original work required training data for each hypothesis
- Has been applied to a range of problems:
  - Modulation classification
  - Image recognition
  - Signal detection (primarily for communications)

Method of Types

- Have observations $r(0), \ldots, r(L\text{-}1)$ from a finite alphabet of values $A = \{a_1, \ldots, a_N\}$.
- A *type* is the pdf estimate generated by the normalized histogram:

$$\hat{P}_r(a_n) = \frac{1}{N} \sum_{l=0}^{L-1} I\big(r(l) = a_n\big),$$

where $I(.)$ is the indicator function.

- For i.i.d. data the probability of a given sequence is

$$\Pr\big[\mathbf{r} = \{r(0), \ldots r(L-1)\}\big] = \prod_{l=0}^{L-1} P_r\big(r(l)\big)$$

Method of Types (cont.)

- This probability can be written as

$$\ln \Pr[\mathbf{r}] = -L\left[H(\hat{P}_r) + D(\hat{P}_r \| P_r)\right]$$

entropy: $$H(P) = -\sum_{n=1}^{N} P(a_n) \ln P(a_n)$$

Kullback-Leibler distance:

$$D(P_1 \| P_0) = \sum_{n=1}^{N} P_1(a_n) \ln \frac{P_1(a_n)}{P_0(a_n)}$$

- K-L distance is nonnegative and zero only when $P_0 = P_1$.
  - The type estimator is the maximum likelihood estimator of P_r.

Detection Algorithm

- Makes no assumptions about probability distributions
- Provably optimal
- Let $\tilde{\mathbf{r}}$ denote training data of length $\tilde{L}$ with some unknown distribution *P***.**
- Derive a generalized likelihood ratio test to determine if observed data $\mathbf{r}$ of length *L* has distribution *P* or a different distribution *Q.*

$$\ln \square(\mathbf{r}) = \ln \frac{\max_{P,Q} P(\mathbf{r})Q(\tilde{\mathbf{r}})}{\max_{P} P(\mathbf{r})P(\tilde{\mathbf{r}})}$$

Detection Algorithm (cont.)

- A type is the maximum likelihood estimate of the probability distribution.
- Substituting types into the likelihood ratio and simplifying yields

$$\ln \Lambda(\mathbf{r}) = LD\left(\hat{P}_r \middle\| \hat{P}_{r,\tilde{r}}\right) + \tilde{L}D\left(\hat{P}_{\tilde{r}} \middle\| \hat{P}_{r,\tilde{r}}\right)$$

- K-L distances are small when the training and observed data are from same distribution, large otherwise.
- Define M_0 to be model where distributions match, and M_1 to be where they do not.

Detection Algorithm (cont.)

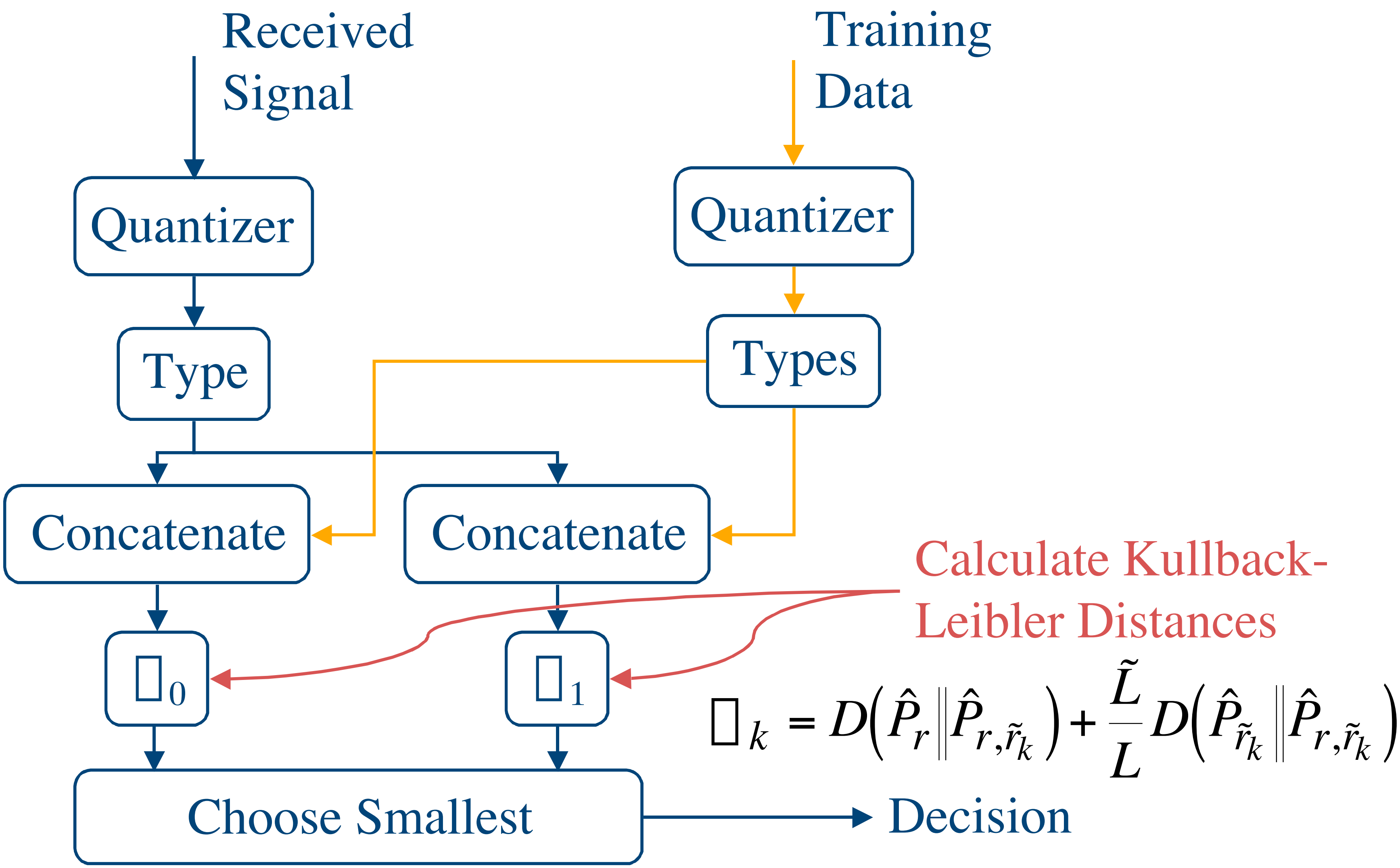
- The decision rule becomes

$$\frac{1}{L}\ln\square(\mathbf{r}) \underset{M_0}{\overset{M_1}{\gtrless}} \square$$

- Can show that
  - False-alarm probability given by $\displaystyle \lim_{L\square} \frac{1}{L} P_{FA} \;\square\;\square\square$
  - Among all training data-based tests with this P_{FA}, this one has the minimum probability of miss.
- Two-sided GLRT:

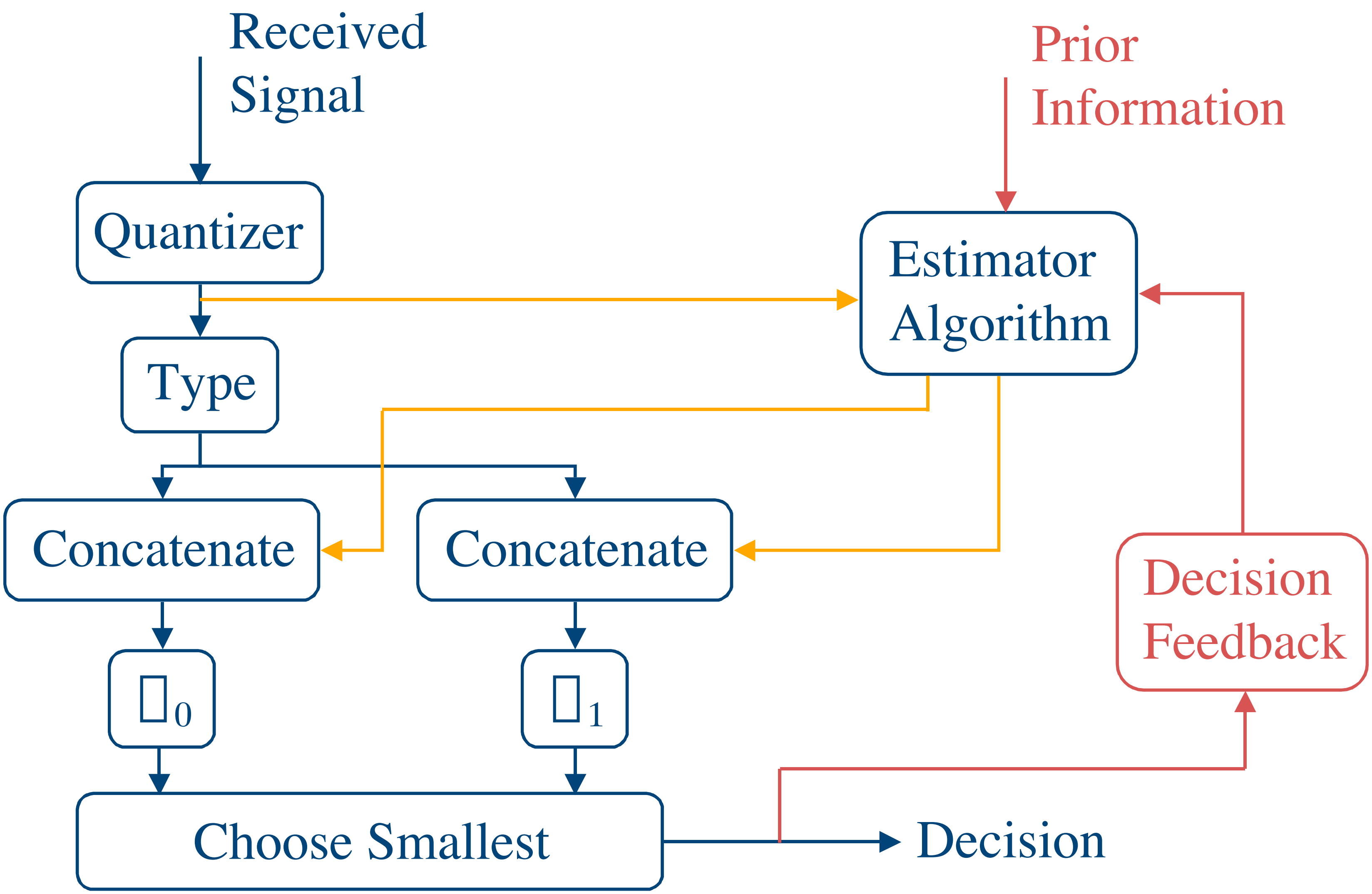
$$D\left(\hat{P}_r \middle\| \hat{P}_{r,\tilde{r}_0}\right)+\frac{\tilde{L}}{L}D\left(\hat{P}_{\tilde{r}_0} \middle\| \hat{P}_{r,\tilde{r}_0}\right) \underset{M_0}{\overset{M_1}{\gtrless}} D\left(\hat{P}_r \middle\| \hat{P}_{r,\tilde{r}_1}\right)+\frac{\tilde{L}}{L}D\left(\hat{P}_{\tilde{r}_1} \middle\| \hat{P}_{r,\tilde{r}_1}\right)$$

Typical Empirical Detector



$$\square_k = D\left(\hat{P}_r \middle\| \hat{P}_{r,\tilde{r}_k}\right) + \frac{\tilde{L}}{L} D\left(\hat{P}_{\tilde{r}_k} \middle\| \hat{P}_{r,\tilde{r}_k}\right)$$

‘Training-Free’ Empirical Detector



Where to from here?

- Goals
  - Design empirical detectors for STAP application
  - Use prior knowledge to make ‘training-free.’
  - Computationally efficient, data-efficient, versatile, and data-driven
- Questions
  - What information is most useful for estimating types directly from test data?
  - How does overall performance compare?